

G. A. Mesyats, A. M. Iskol'dskii, V. V. Kremnev, L. G. Bychkova, and Yu. I. Bychkov  
 Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 3, pp. 77-81, 1968

It has been shown [1-3] that the discharge buildup time  $\tau$  for initiation by single electrons substantially exceeds the time expected from the one-avalanche streamer mechanism in the case of gaps about 1 mm long breaking down in  $10^{-9}$  sec or so. This indicates that the mechanism involves many avalanches. If the discharge is initiated by many electrons ( $10^4$ ), the current rise is due to avalanche multiplication of initiating electrons [4]. Then  $\tau$  equals the time for an avalanche to build up to about  $10^8$  electrons [5].

Little is known about electron multiplication in one-electron initiation, except that a diffuse glow occurs throughout the gap during the initial buildup, together with narrow channels of low luminosity [6], while the stage of rapid current rise can be ascribed to avalanche multiplication of secondary electrons [4]. Here we consider electron multiplication in the primary process, and also the mechanism of secondary-electron extraction in one-electron initiation in strong fields.

**1. Electron spread in an avalanche.** An electron appearing at the cathode leads to production of an electron avalanche. If the field is strong, many electrons are formed in the avalanche as it advances a distance much less than the gap width. The field produced by the electrons and ions begins to influence the behavior of the avalanche. The ion field retards the tail of the electron avalanche, while the electron field will accelerate the electrons at the head, i.e., the electrons begin to spread out. This effect appears even during the exponential growth of the avalanche.

The electron density  $n_e^*$  and ion density  $n_i^*$  are as follows in a cylindrical coordinate system  $r, z$ :

$$n_e^* = \pi^{-1/2} r_d^{-3} \exp(\alpha_0 v_- t - [r_2 + (z - v_- t)^2] r_d^{-2}), \quad (1.1)$$

$$n_i^* = \alpha_0 v_- \int_0^t n_e^*(t') dt', \quad r_d = \sqrt{4Dt}. \quad (1.2)$$

Here  $r_d$  is the diffusion radius,  $D$  is diffusion coefficient,  $\alpha_0$  is the coefficient of collisional ionization, and  $v_-$  is electron drift velocity.

The following are [7] the electron and ion fields in an avalanche as functions of  $z$ :

$$E_e^z = -E_0 M(y), \quad E_i^z = E_0 a e^{a y} \int_y^\infty e^{-ax} M(x) dx, \\ E_0 = \frac{q_e \exp(\alpha_0 v_- t)}{4\pi \epsilon_0 r_d^2}, \quad y = \frac{z - v_- t}{r_d}, \quad a = \alpha_0 r_d, \\ M(y) = \frac{\text{erf}(y)}{y^2} - \frac{2}{\pi} \frac{\exp(-y^2)}{y}, \quad (1.3)$$

in which  $q_e$  is electron charge.

We normalize the expressions for the fields and space-charge densities:

$$n_e = \frac{n_e^*}{\pi^{-1/2} r_d^{-3} \exp(\alpha_0 v_- t)}, \quad E_e = \frac{E_e^z}{E_0}, \\ n_i = \frac{n_i^*}{\pi^{-1/2} r_d^{-3} \exp(\alpha_0 v_- t)}, \quad E_i = \frac{E_i^z}{E_0}. \quad (1.4)$$

Figure 1 shows  $n_e, n_i, E_e,$  and  $E_i$  as functions of  $(z - v_- t)/r_d$ .

To the right of the point where  $E_e = E_i$  we have  $E_e > E_i$ . The field  $E_e + E_i$  is added to the external field and will tend to detach electrons from the head. The sum field is reduced to the left of  $E_e = E_i$  and the electrons are retarded. Consider the number of electrons in a field ex-

ceeding the external field, for which we determine the surface at which  $E_e + E_i = 0$  and take the integral over the entire volume in the direction of increasing  $r$  an  $z$ . For approximate purposes we restrict consideration to determination of the surface at which the projection of  $E_e + E_i$  on the  $z$ -axis is zero.

We assume the distribution of  $n_i$  and  $n_e$  to be spherically symmetrical. The center of density for the ions lies at  $y = -\beta$ , where  $\beta$  is defined by

$$E_i(-\beta, a) = 0.$$

Then for the critical surface whose coordinates are  $\rho(y)$  we have

$$\frac{y + \beta}{\sqrt{(y + \beta)^2 + \rho^2}} E_i \left[ \sqrt{(y + \beta)^2 + \rho^2}, a \right] + \\ + \frac{y}{\sqrt{y^2 + \rho^2}} E_e(\sqrt{y^2 + \rho^2}) = 0 \quad \left( \rho = \frac{r}{r_d} \right). \quad (1.5)$$

Numerical solution of (1.5) for  $a = 1$  and  $2$  shows that  $\rho(y)$  may be approximated by a parabola:

$$\rho = 2.4(y - b)^{0.5}, \quad (1.6)$$

in which  $b = 0.495$  for  $a = 1$  and  $b = 0.645$  for  $a = 2$ .

The proportion of electrons within the paraboloid is

$$N^{(1)} = \exp(-\alpha_0 v_- t) \int_{z_0}^\infty \int_0^{r(z)} n_e 2\pi r dr dz,$$

where  $z_0$  is deduced from (1.6) with  $\rho = 0$  and  $r(z)$  from (1.6) with the substitution

$$\rho = r/r_d, \quad y = (z - v_- t)/r_d.$$

The  $N^{(1)}$  for  $a = 1$  and  $2$  are, respectively,  $0.18$  and  $0.13$ .

The number  $N$  of electrons in the avalanche increases as time passes, while  $\alpha$  decreases and  $r_d$  increases. Calculations [7-9] were made for  $a = \alpha r_d$  at  $p = 760$  mm Hg and  $E/p = 150$  V/cm-mm Hg in order to estimate  $N^{(1)}$ . As  $\alpha$  increased from  $0.7\alpha_0$  to  $0.98\alpha_0$  there was about a 10% increase in  $a$ , and Fig. 1 shows that this leads to increase in  $N^{(1)}$ . This result agrees with measurements of avalanche radius in vapors of

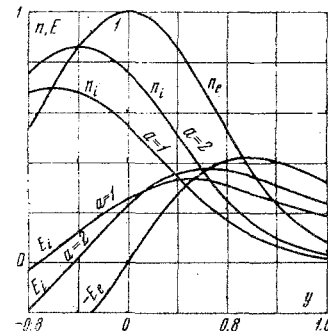


Fig. 1. Normalized  $n_e, n_i, E_e,$  and  $E_i$  at the  $z$ -axis near the center of the electron cloud.

organic liquids [10], but these data indicate that the avalanche radius is greater than  $r_d$  for  $N \geq 5 \cdot 10^7$  on account of Coulomb repulsion between the electrons. This reduces  $E_e$  and correspondingly reduces the electron spread.

This means that more than 10% of the electrons will be moving with a velocity exceeding  $v_*$  even at the start of retardation of the avalanche by  $E_i$ , while the rest will move at less than this velocity.

**2. Avalanche chains.** When  $E_i$  becomes comparable with the external field, the electron head becomes detached from the avalanche and starts to form a new avalanche, and so on. The proportion of electrons lost from the avalanche will be less than the  $N(i)$  derived in the previous section because only electrons whose speed considerably exceeds  $v_*$  will be lost.

Consider the total number of electrons in such a chain. We neglect overlap between the electron and ion clouds, and we also assume that the avalanche broadens by free diffusion. If we also assume that the avalanche grows exponentially with  $\alpha = \text{const}$ , the following is the number of electrons in the avalanche for one initiating electron when  $E_i = E$ :

$$N_0 = \frac{16\pi\epsilon_0 u_T \mu_+ \ln N_0}{q_e \alpha}, \quad (2.1)$$

in which  $u_T$  is the thermal velocity of the electrons.

Electrons detached from the head move in the field

$$E^* = k_E E \quad (k_E > 1),$$

since the field of the electron cloud will be superimposed on the external field  $E$ . The number of avalanches in a path  $z$  is

$$z/z_k = \alpha \ln N_0 / N^*,$$

in which  $z_k$  is avalanche length and  $N^*$  is the number of electrons ejected from the avalanche. Then the total number of electrons in such a chain is

$$N_1 = N_0 \frac{z}{z_k} \quad \text{or} \quad N_1 = \frac{16\pi\epsilon_0 u_T z^2}{q_e}. \quad (2.2)$$

The conductivity of an avalanche chain across the gap ( $z = \delta$ ) is  $N_1 \mu - q_e \delta^{-2}$ . It is found [8,9] for nitrogen at  $E/p > 10^2$  V/cm-mm Hg that  $u_T \approx 0.3(E/p)^{0.62}$ , i.e., the conductance is  $\sim 2.5 \cdot 10^{-6} (p\delta)^{-1}$  ohm $^{-1}$ .

Chains of avalanches such as those described above should take the form of thin weakly luminescent channels at the stage at which there is still no potential drop in the gap; such channels would appear to have been observed [6].

In fact, narrow channels within about  $10^{-9}$  sec acquired a diameter of about  $10^{-2}$  cm and crossed the gap in not more than  $10^{-9}$  sec for  $\delta = 0.4$  cm,  $p = 46$  mm Hg, and  $E/p = 1.43 \cdot 10^3$  V/cm-mm Hg.

An avalanche chain under such conditions should have a diameter of about  $d_1 \approx 2\sqrt{6Dt}$  and should bridge the gap in a time  $t_1 \approx \delta/k_v v_*$ , in which  $k_v > 1$  takes account of the increase in the speed of the avalanche chain relative to  $v_*$ . If the electrons are ejected for  $E_i = E$  we have  $k_v \approx 2$ . For nitrogen [8,9]

$$D \approx 2/3 u_T \mu_+ \approx 3.10^4 \text{ cm}^2 \text{ sec}^{-1},$$

so we get

$$d_1 \approx 3 \cdot 10^{-2} \text{ cm} \quad t_1 \approx 1.5 \cdot 10^{-9} \text{ sec}.$$

These agree with observed values in order of magnitude [6].

Secondary electrons are produced by photons from the primary avalanches. If we assume that these photons are emitted by excited molecules with a mean lifetime  $\tau_b$ , the time of discharge buildup should be of the order of  $\tau_b$ , as is observed [3,12].

The concept of avalanche chains is confirmed by the presence of many channels in the final stage of breakdown. An avalanche chain is of low conductivity and is quasi-neutral, so the time of spark voltage drop related to avalanche electron multiplication should not be dependent on the number of original initiating electrons, as is found [4].

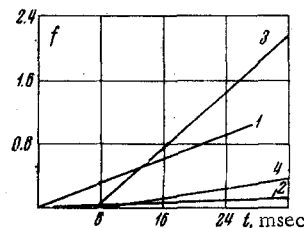


Fig. 2. Change in photoelectric emission from copper for  $\delta = 1$  mm,  $p = 760$  mm Hg, and  $U$  (kV) of: 1) 30, clean electrodes, 2) 30, after 4000 sparks, 3) 10, clean electrodes, spark illumination, 4) 10, after 4000 sparks, spark illumination.

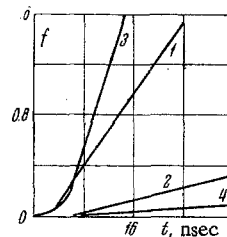


Fig. 3. Field emission from copper with  $\delta = 1$  mm at  $U = 30$  kV and  $p$  (mm Hg) of: 1) 20, clean electrodes, 2) 760, clean electrodes, 3) 20, after 4000 sparks, 4) 760, after 4000 sparks.

**3. Secondary processes.** The distribution in the discharge delay was examined in relation to the surface state of the cathode in order to investigate the role of the cathode in the secondary process. The method of measurement was as previously described [3]. We plotted  $f(t) = \ln n_t/n_0$ ,  $f(t_1) = 1$  ( $n_t$  is the number of discharges with a delay of  $t$  or more, and  $n_0$  is the total number of discharges). If  $t_1 \gg 10^{-9}$  sec, the  $f(t)$  curves become straight lines. Copper and tungsten cathodes gave a marked dependence of  $t_1$  on the number of previous discharges. Numerous measurements of  $t_1$  showed that  $t$  increases with the number of sparks. The effect is slight for the first few hundred sparks, but it becomes marked for  $n > 10^3$ . These effects are not observed for aluminum cathodes [13].

There are two reasons for increase in  $t_1$ : 1) the photoelectric emission from the cathode may deteriorate, which reduces the effects of secondary processes, 2) if the discharges reduce the field emission from the cathode, this will reduce the electron current  $i_0$  that initiates the breakdown and thus will increase  $t_1$ , because  $t_1 \propto 1/i_0$  for  $E$ ,  $p$ , and  $\delta$  constant.

The following tests were performed to establish which of these two effects governs  $t_1(n_0)$ . With  $E = 3 \cdot 10^5$  V/cm,  $\delta = 0.1$  cm,  $p = 760$  mm Hg (air), and clean copper electrodes, we measured the delay and drew up  $f(t)$  curves. The same electrodes were then used with the same  $p$  and  $\delta$  at  $E = 10^5$  V/cm with illumination from an auxiliary spark via a quartz window (curves 1 and 3 in Fig. 2). Then 4000 sparks were passed, followed by recordings with and without the spark illumination (curves 2 and 4 in Fig. 2).

Curves 1 and 2 are to be compared with 3 and 4, which characterize the photoelectric emission. Line 4 is much less steep than line 3, which shows that the cathode emissivity is much reduced by 4000 sparks. The  $f(t)$  for low pressures similarly serve to characterize the change in field emission in response to a number of discharges.

It has been found [3] that for  $\delta \leq 0.1$  cm at atmospheric pressure there is a certain probability  $P < 1$  of electron emission from the cathode even at high overvoltages. The ratio  $E/p$  had to be increased to produce  $P \approx 1$ . We therefore recorded two distributions at  $E = 3 \cdot 10^5$  V/cm for clean copper electrodes, one at 20 mm Hg and the other at 760 mm Hg (curves 1 and 2 in Fig. 3). The curves were again recorded after 4000 sparks (curves 3 and 4). As  $P = 1$  implies that  $i_0 \approx j_e/t_1$  for the electron current from the cathode that initiates breakdown, the

slopes of the  $f(t)$  curves for 20 mm Hg (1 and 3 in Fig. 3) characterize  $i_0$  at the start and end. The sparking tends to increase the field emission [14].

Photoelectric emission from the cathode is thus an important secondary process in the production of discharges in the nanosecond range.

#### REFERENCES

1. Yu. E. Nesterikhin, V. S. Komel'kov, and E. Z. Meilikhov, "Impulse breakdown of short gaps in the nanosecond range," *Zh. tekh. fiz.*, **34**, no. 1, 40-52, 1964.
2. G. A. Mesyats, Yu. P. Usov, and G. S. Korshunov, "Breakdown delay in gaps under irradiation for use in nanosecond techniques," *Radiotekhnika i Elektronika*, **9**, no. 5, 882-887, 1964.
3. G. A. Mesyats and Yu. I. Bychkov, "A statistical study of breakdown delay in short gas gaps at very high electric fields in the nanosecond range," *Zh. tekh. fiz.*, **37**, no. 9, 1712, 1967.
4. G. A. Mesyats, Y. J. Bichkoff, A. M. Iskoldski, "On the increase by spark current during pulse breakdown of air gaps in nanosecond time range," *Proc. 8-th Conf., on Phenomena in Ionized Gases*, Vienna, p. 210, 1967.
5. R. C. Fletcher, "Impulse breakdown in the  $10^{-9}$  sec range of air at atmospheric pressure," *Phys. Rev.*, vol. 76, no. 10, pp. 1501-1511, 1949.
6. V. V. Vorob'ev and A. M. Iskoldskii, "Impulse breakdown in a uniform field in air at substantial overvoltages," *Zh. tekh. fiz.*, **36**, no. 11, 2095-2098, 1966.
7. K. J. Schmidt-Tiedeman, "Die Raumladungsbremmung von Elektronenlawinen," *Zs. Naturf.*, vol. 14a, p. 989, 1959.
8. H. Schlumbohm, "Messung der Driftgeschwindigkeiten von Elektronen und positiven Ionen in Gasen," *Zs. Phys.*, vol. 182, p. 317, 1965.
9. H. Schlumbohm, "Stoßionisierungskoeffizient  $\alpha$ , mittlere Elektronenenergien und die Beweglichkeit von Elektronen in Gasen," *Zs. Phys.*, vol. 184, p. 492, 1965.
10. K. Richter, "Die Eigenschaften von Elektronenlawinen bei hohen Verstärkungen in Äther," *Zs. Phys.* vol. 180, p. 489, 1964.
11. H. Raether, *Electron Avalanches and Breakdown in Gases*, London, 1964.
12. H. Tholl, "Der Übergang der Elektronenlawine in den Kanalaufbau bei hoher Überspannung in Stickstoff (mit geringen  $\text{CH}_4$ -Zusatz)," *Zs. Naturf.*, vol. 18a, no. 5, p. 587, 1963.
13. Yu. I. Bychkov and L. G. Bychkova, "Secondary processes in impulse breakdown of air gaps," *Electronic Techniques, series 3: Gas-Discharge Devices [in Russian]*, no. 2, 39-44, 1967.
14. D. Meek and G. Craggs, *Electrical Breakdown in Gases [Russian translation]*, Izd-vo inostr. lit., 1960.

5 November 1967

Tomsk